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EFFECT OF RANDOM ERRORS IN WIGGLER MAGNETIC FIELDS  
ON FREE-ELECTRON LASER GAIN AND OUTPUT

by

Dong Zhiwei, Tian Shihong, Yang Zenhua



19951023 065

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**HUMAN TRANSLATION**

NAIC-ID(RS)T-0364-95

11 Sep 95

MICROFICHE NR: 95C000569

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English pages: 9

Source: Qiangjiguang Yu Zizishu, Vol. 4, Nr. 2, May 1992;  
pp. 299-304

Country of origin: China

Translated by: Edward A. Suter

Requester: NAIC/TATD/Dr. James Newton

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NAIC- ID(RS)T-0364-95

Date 11 September 1995

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## EFFECTS OF RANDOM ERRORS IN WIGGLER MAGNETIC FIELDS ON FREE-ELECTRON LASER GAIN AND OUTPUT

(Translation of Cichang Suiji Wucha Dui Ziyoudianzi Jiguang  
Zengyi He Shuchu De Yingxiang)

*Dong Zhiwei, Tian Shihong, Yang Zhenhua*

(Institute of Applied Physics and Computational Mathematics,  
P.O. Box 8009, Beijing, 100088)

**Abstract:** [In this article,] the small signal gain and exponential gain of the light field when there are random errors in the magnetic field are deduced based on the free-electron laser (FEL) longitudinal mode unified theory<sup>[1]</sup> of Professor Yu Min. In addition, by combining [theory with] the parameters of the SG-1<sup>2</sup> device, the scaling relationship between magnetic field errors in the areas of exponential gain  $\sigma$  and gain parameter  $g_0$  is given. Moreover, the three-dimensional WAGFEL program is used to simulate the effects of magnetic field random errors on free-electron laser gain and saturation gain. Simulations verify the conclusions of linear theory.

**Key terms:** magnetic field random error, exponential gain, saturation output power

### I. Introduction

The elimination of the effects and detrimental factors of random errors in wiggler magnetic fields on free-electron laser output has been a popular topic of free-electron laser research recently, and many articles have been published about it<sup>[2-3]</sup>. Generally speaking, the existence of wiggler magnetic field random errors causes additional random vibrations to appear in the electron beam, leads to irregular changes in phase in the

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<sup>1</sup> Numbers on the extreme right represent the pages of the original document.

<sup>2</sup> In the English abstract, the authors refer to this device as the SG-1. It can also be referred to as *Shuguang*-1, its romanized name, or "Aurora-1," its name in translation.

ponderomotive potential well, causes declining captured electrons<sup>3</sup> to increase, and thus brings about a decrease in the FEL's output gain and light beam quality, which threatens to cause a loss of coherence.

This article gives the effects of magnetic field random errors on free electron laser gain under cold beam conditions according to the unified longitudinal mode FEL theory of Professor Yu Min, and provides a detailed discussion of exponential gain. Finally, three-dimensional simulation confirms the results of linear theory.

## II. Expressions of gain where magnetic field random error exists in FELs

According to Professor Yu Min's FEL longitudinal mode theory, the evolution equation of light fields can be written as

$$\frac{da_n(z)}{dz} = -i \frac{\omega_b^2}{4\pi} \frac{\Delta\omega}{\omega_n} \int_{-\frac{\pi}{\Delta\omega}}^{\frac{\pi}{\Delta\omega}} dt_0 \frac{\beta_1(z, t_0)}{\beta_1(z, t_0)} \beta_1^{(0)} \cdot \exp\{-ik_n z + i\omega_n t_0\} \frac{f(\gamma_0, t_0)}{n_0} \exp\{ik_n \int_0^z \frac{dz}{\beta_1(z, t_0)}\} \quad (1)$$

where  $a_n(z)$  is the Fourier transform of the complex light field's  $n$ th mode.

Assuming that the beam is a cold beam, and not considering effects brought about by sideband, additional high-light fields, etc., formula (1) can further be written as

$$\frac{da_n(z)}{dz} = -ij \int_{-\frac{\pi}{\Delta\omega}}^{\frac{\pi}{\Delta\omega}} dt_0 H_n(z) e^{i\omega_n t_0} \exp\{ik_n \int_0^z dz \Delta\beta_1^{(2)}\} \quad (2)$$

$$H_n(z) = \sin k_n z e^{i\psi}$$

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where  $\psi$  is the ponderomotive potential phase of the electron (here, it is assumed that the wiggler is a linearly polarized field of a parabolic polar plane focus).  $\Delta\beta_{//}^{(2)}$  is the longitudinal electron velocity brought about by  $a_n$

$$\Delta\beta_1^{(2)} = \frac{\mu^2}{\gamma_0^3} \int_0^z dz \frac{d\Delta\gamma^{(2)}}{dz} \quad (3)$$

Under the assumptions of linear theory, we can obtain

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<sup>3</sup> Best guess for "tui buhuo dianzi."

$$\frac{da_n(z)}{dz} = iQH_n^*(z) \int_0^z dz \int_0^z a_n(z) H_n(z) \quad (4)$$

$$Q = \frac{\omega_p^2 a_v^2 \mu^2 k_n}{2\gamma^3 c^2}$$

Below, we concretely derive  $H_n(z)$  when random magnetic field error exists, under conditions where sideband and betatron effects are not taken into account

$$\Delta\beta_{//}^{(0)} = \frac{a_v^2}{2\gamma_0^2} \cos 2k_n z - \frac{a_v^2}{2\gamma_0^2} (x_p^2 k_x^2 + y_p^2 k_y^2) + \frac{a_v}{\gamma} \delta\beta_x \sin k_n z - k_p^2 \delta x^2 - \delta\beta_x^2 \quad (5)$$

The third, fourth, and fifth terms in formula (5) are the latitudinal electron positions and velocity vibrations caused by random errors in the magnetic field. The second term, however, represents the effect of electronic emission. In the following calculations, we will omit this term.

By substituting for  $\Delta\beta_{//}^{(0)}$  and  $H_n(z)$  and considering that there can only be several modes that can resonate with the electron beam and the system parameters, we obtain

$$\begin{aligned} \frac{da_n(z)}{dz} = & i \frac{Q}{4} [J_n(A_n) - J_{n+1}(A_n)]^2 e^{-i\Delta k z} \exp\{ik_n k_p^2 \int_0^z \delta x^2 dz + ik_n \int_0^z \delta\beta_x^2 dz\} \\ & \cdot \int_0^z dz \int_0^z da_n(z) e^{i\Delta k z} \exp\{-ik_n k_p^2 \int_0^z \delta x^2 dz - ik_n \int_0^z \delta\beta_x^2 dz\} \end{aligned} \quad (6)$$

The average contribution of a group of wiggler magnets to electron beam deviation can be expressed as

$$\begin{cases} \langle \delta x^2 \rangle = \left( \frac{a_v k_v}{\gamma k_{px}} \right)^2 \sigma^2 \frac{\lambda_v}{2} \left( z - \frac{\sin k_{px} z}{k_{px}} \right) \\ \langle \delta\beta_x^2 \rangle = \left( \frac{a_v k_v}{\gamma k_{px}} \right)^2 \sigma^2 \frac{\lambda_v}{2} \left( z + \frac{\sin k_{px} z}{k_{px}} \right) \end{cases} \quad (7)$$

$\sigma$  is the root-mean-square value of the magnetic field random error.

By substituting (7) into (6), and letting

$$g = \frac{Q}{4} [J_n(A_n) - J_{n+1}(A_n)]^2$$

we can obtain the average effect of magnetic field random error on free-electron laser output

$$\frac{da_n}{dz} = ig e^{-i\Delta k z} \exp\{iDz^2\} \int_0^z dz \int_0^z dz e^{i\Delta k z} a_n(z) \exp\{-iDz^2\} \quad (8)$$

where

$$D = \left( \frac{a_n k_n}{\gamma} \right)^2 \sigma^2 \frac{\lambda_n}{2} k_n$$

### 1. Small-signal gain

Since  $a_n(z) = a_0$  and the light field has no relation to  $z$ ,  $a_n$  can give

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$$G = \ln \left| \frac{a_n(L)}{a_n(0)} \right|^2 = i2g \int_0^L dx \int_0^x dz e^{-i\Delta k(x-z)} \exp\{i(x^2 - z^2)D\} (x - z) \quad (9)$$

Assuming that magnetic field error is very small, the term  $\exp\{iD(x^2 - z^2)\}$  can be expanded to

$$\exp\{iD(x^2 - z^2)\} = 1 + iD(x^2 - z^2) - \frac{D^2}{2} (x^2 - z^2)^2 \quad (10)$$

By substituting formula (10) into formula (9), after many diverse calculations, when it is possible to obtain the approximate magnetic field random error  $\sigma$ , the small-signal gain of free-electron lasers is

$$\begin{aligned} G = G_1 + G_2 + G_3 = & -g_0 \frac{d}{d\theta} \left( \frac{\sin\theta}{\theta} \right)^2 + DL^2 g_0 \frac{d^2}{d\theta^2} \left( \frac{\sin\theta}{\theta} \right)^2 \\ & + \frac{D^2 L^4}{2} g_0 \left\{ \frac{d^3}{d\theta^3} \left( \frac{\sin\theta}{\theta} \right)^2 + \frac{1}{\theta^4} [\sin 2\theta (2\theta^3 - 3\theta) - 6\sin^2\theta] \right\} \end{aligned} \quad (11)$$

where  $\theta = \Delta k L / 2$ , and, in areas of practical interest,  $G_2$  tends towards zero.

### 2. Exponential gain

Since light field variation is considered to follow  $a_n = a_0 \exp(\Gamma z)$ , formula (8) can be converted to

$$\frac{a_s(L) - a_s(0)}{a_s(0)} = ig_0 \int_0^L dx \int_0^x dz e^{-i\Delta k(x-z)} \exp\{i(x^2 - z^2)D\} (x-z) e^{\Gamma z} \quad (12)$$

Similarly, by expanding  $\exp\{iD(x^2 - z^2)\}$ , it is easy to obtain

$$\Gamma(\Gamma + i\Delta k)^2 = g + \frac{8LgD}{(\Gamma + i\Delta k)} \quad (13)$$

By comparing the small-signal gain and exponential gain<sup>[1]</sup> of formulas (11) and (13) with that of free-electron lasers when  $\delta B = 0$ , it is not difficult to discover that when  $D = 0$ , (11) and (13) degenerate into the original corresponding expressions. But the existence of  $\delta B$  causes the gain values of free-electron lasers under corresponding gain to decline. Based on our calculations and analyses, magnetic field random errors have the same effect on free-electron laser gain that electron beam quality (for example, beam dispersion, emission intensity, etc.) does. Both have a marked effect in small-signal gain areas, but their effect shrinks in exponential gain areas.

### III. Effects of magnetic field random errors on exponential gain of free-electron lasers

We did calculations of three different groups of device parameters from formula (13). The results are shown in the following tables.

Table 1

$\sigma / \%$	$\alpha / \text{cm}^{-1}$	$ \alpha - \alpha_0  / \%$	$\Gamma / (\text{dB} \cdot \text{m}^{-1})$
0.0	0.46970	0.0	20.00
1.0	0.04530	3.55	19.70
2.0	0.04197	10.6	18.50

Calculation condition :  $g = 0.000345$ ,  $\Delta k = 0.01047$ ,  $DL = 109.21\sigma^2$

Tables 1 through 3 [demonstrate] the effects of magnetic field random errors on exponential gain of free-electron lasers under different device parameter conditions. Here, the beam and system parameters of the SG-1 device are close to the conditions given in Table 1.



Table 2

$\sigma / \%$	$\alpha / \text{cm}^{-1}$	$ \alpha - \alpha_0 / \alpha_0  / \%$
0.0	0.24	0.0
1.0	0.2395	0.2
2.0	0.2379	0.86
3.0	0.2353	1.96
5.0	0.2258	5.9
7.0	0.2067	13.89
10.0	0.1800	25.0

Calculation condition :  $g = 0.0213$ ,  $\Delta k = 0.0$ ,  
 $DL = 109.21\sigma^2$

Table 3

$\sigma / \%$	$\alpha / \text{cm}^{-1}$	$ \alpha - \alpha_0 / \alpha_0  / \%$
0.0	0.6874	0.0
1.0	0.6864	0.07
2.0	0.6853	0.3
5.0	0.6743	1.9

Calculation condition :  $g = 0.50$ ,  $\Delta k = 0.0$ ,  
 $DL = 109.21\sigma^2$

From the above data, we can easily find that even for the same wiggler device, the effects of magnetic field random errors give different results according to changes in beam quality. Because the electron beam quality of the SG-1 device was not up to expectations, demands on wiggler magnetic field error are necessarily very strict.

In addition, the SG-1 device tends towards low-gain free-electron-laser amplifiers. In a sense, it depends on length<sup>4</sup> to reach saturation of high output power. Therefore, the effect of magnetic field random error on saturation output power is more obvious than its effect on exponential growth (or, from another point of view, magnetic field random error has a greater effect in nonlinear areas than in linear areas.)

#### IV. Numerical simulation results

In the revised version of the WAGFEL program, we considered the effects of random errors in magnetic fields on free electron laser output.<sup>[6]</sup> We carried out calculation simulations of the condition of the amplifier under SG-1 device parameters.

Figure 1 shows the development of free-electron laser saturation output power plotted against random errors in magnetic fields.

Here, the magnetic field random error  $\sigma$  divided by 3 is approximately equal to the root-mean-square of the wiggler magnetic field random error estimated in the experiment.

Figure 2 shows the development of free-electron-laser output power when given different  $\sigma$ .

<sup>4</sup> Best guess for "...ta kao pinchangdu..."

From Figure 2, it is easy to see that even when there are corresponding magnetic field random errors, there is not a very large decline in the laser's exponential gain. For example, a comparison of curves 1 and 2 shows that exponential gain falls only 2dB/m, from 18.3 to 16.1 dB/m. However, there is a very sharp decline in the laser's saturation output power ( $P_s$  falls from 78 to 37MW). The large decline in saturation output power is, for one thing, due to a fall in exponential gain in a long linear area, and, for another, caused by the nonlinear effect. To explain this point, in Figure 3, we plot the relationship between exponential gain and saturation output power brought about by different magnetic field errors under SG-1 parameters. It is easy to tell from this figure that tiny changes in  $G$  bring about large-scale declines in  $P_s$ . We can also get a clearer idea about the second point from a comparison of curves (1) and (2) in Figure 2. There is not much difference in the power output of curve (1) ( $\sigma = 5\%$ ) and curve (2) ( $\sigma = 10\%$ ) when the linear area ends, but there is a difference of greater than 50% between their saturation output powers. This shows that in nonlinear areas, magnetic field error has a more significant effect on output. This is consistent with theoretical analysis of nonlinear areas.

To make a comparison with nonlinear theory, we took the most probable values of output magnetic field random error variations from the simulation results and listed them in Table 4.

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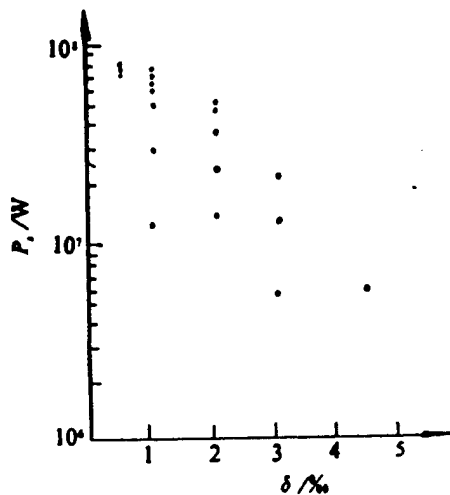


Figure 1

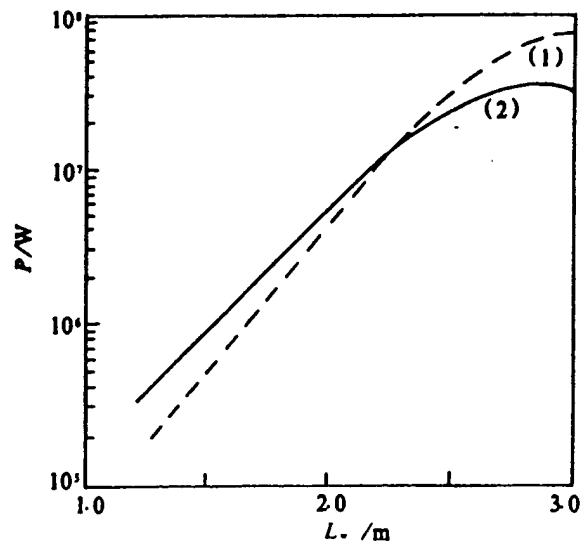


Figure 2

Figure 1. FEL saturation output power  $P_s$  as a function of random errors  $\sigma$  of wiggler magnetic field  $B$ , where random arrangement of  $\delta B$  causes the values of  $P_s$  to be different [even] when paired with the same  $\sigma$ .

Figure 2. FEL output power as a function of wiggler length where there are different wiggler field random errors. For curve (1),  $\sigma = 5.0\%$ , and for curve (2),  $\sigma = 10.0\%$ .

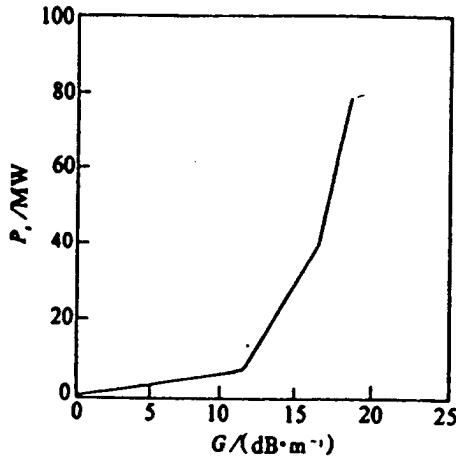


Figure 3

Figure 3. Saturation power  $P_s$  as a function of exponential gain  $G$  caused by different magnetic field random errors, based on SG-1 parameters.

Table 4. From the data in the table above, we can determine that the results of simulation calculations in the linear section are fairly close to the results of linear theory. A quantitative comparison cannot yet be made, because there were too few points calculated by linear theory and simulation results were not sufficient to cause large statistical fluctuations.

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## V. Conclusions

(1) Random errors in magnetic fields have clearly detrimental effects on the output of free-electron lasers. Also, these effects intensify as the quality of the beam deteriorates, and are related to the system's gain parameter  $g_0$ .

(2) The results of our simulation calculations with SG-1 device design parameters showed that when the root-mean-square value of magnetic field random errors is less than

2‰, magnetic field errors have no effect on the output of free-electron laser amplifiers. As magnetic field random errors increase, free electron laser output drops distinctly, and random fluctuations increase by a large margin. This is bound to cause the experiment's efficiency-to-cost ratio to decrease.

(3) The effect of magnetic field random errors on small-signal gain areas is larger than their effect on exponential gain areas. Their effect on nonlinear areas near saturated points is also far greater than their effect on exponential gain areas.

(4) The above simulation calculations were carried out on the basis of erasing electronic rapid change oscillation motion, and did not strictly consider beam transmission. Therefore, it was not possible to carefully consider the effects of magnetic field errors on beam transmission. But from the point of view of the electrons' betatron orbits, the effects of magnetic field random errors at the wiggler exit point are quite evident, and almost cause the electron beam to "run into a stone wall."

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**Note:** The original of this document was received on July 3, 1991, and the corrected copy was received on January 3, 1992.

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